

NON-LOCAL INTERACTIONS FOR THE DEUTERON USING SPHERICAL BESSEL FUNCTIONS

R. MEHREM[★]

Associate Lecturer

The Open University in the North West

351 Altrincham Road

Sharston, Manchester M22 4UN

United Kingdom

[★] Email: rami.mehrem@btopenworld.com.

ABSTRACT

Non-local interactions are assumed for the deuteron with form factors

$g_C(k) = j_0(b_1 k) j_0(b_2 k)$ for the central part responsible for the S-state and

$g_T(k) = j_1(b_1 k) j_1(b_2 k)$ for the tensor part responsible for the D-state, where b_1 and b_2 are range parameters for the proton and neutron, respectively. The analytically obtained wavefunctions in coordinate space have different forms in three different regions. The inner most region is between $r = 0$ and $r = b_2 - b_1$ (assuming $b_2 > b_1$), followed by a region between $r = b_2 - b_1$ and $b_1 + b_2$ and finally the region $r > b_1 + b_2$. The resulting wavefunctions and their derivatives are found to be continuous at the boundaries. The ensuing calculations are simplified by setting $b_1 = b_2 = b$. Good agreement is obtained for the quadrupole moment. Neutron-proton scattering calculations will follow in the next communication.

1. Introduction

Non-local interactions for the neutron-proton scattering and bound state (the deuteron) are written in the form

$$V_C(k, k') = -\frac{\lambda_C}{M} g_C(k) g_C(k'), \quad (1.1)$$

and

$$V_T(k, k') = \frac{\lambda_T}{M} g_T(k) g_T(k'), \quad (1.2)$$

where $V_C(k, k')$ is the central part responsible for the S-State of the deuteron and $V_T(k, k')$ is the tensor part corresponding to the D-state of the deuteron. λ_C and λ_T are the strengths of the central and tensor parts of the potential, respectively and are assumed positive. The form factors $g_C(k)$ and $g_T(k)$ are assumed to be of the form

$$g_C(k) = j_0(b_1 k) j_0(b_2 k), \quad (1.3)$$

and

$$g_T(k) = j_1(b_1 k) j_1(b_2 k), \quad (1.4)$$

where $j_l(bk)$ is a spherical Bessel function of order l , b_1 and b_2 are range parameters for the proton and neutron, respectively. The deuteron (reduced) S- and D- wavefunctions in momentum space, $u(k)$ and $w(k)$, respectively are then [1,2]

$$u(k) = A \sqrt{\frac{2}{\pi}} \frac{j_0(b_1 k) j_0(b_2 k)}{k^2 + \alpha^2} \quad (1.5)$$

and

$$w(k) = B \sqrt{\frac{2}{\pi}} \frac{j_1(b_1 k) j_1(b_2 k)}{k^2 + \alpha^2}, \quad (1.6)$$

where A and B are the normalisation constants for the S- and D- wavefunctions, respectively and $\alpha^2 = M|E_d|$, M is mass of the nucleon, E_d is the energy of the deuteron.

Using

$$u(r) = \sqrt{\frac{2}{\pi}} r \int_0^\infty k^2 dk u(k) j_0(kr), \quad (1.7)$$

$$w(r) = \sqrt{\frac{2}{\pi}} r \int_0^\infty k^2 dk w(k) j_2(kr), \quad (1.8)$$

and assuming $b_2 \geq b_1$,

(i) for $r \leq b_2 - b_1$

$$u(r) = A i_0(\alpha b_1) k_0(\alpha b_2) (\alpha r) i_0(\alpha r), \quad (1.9)$$

$$w(r) = B i_1(\alpha b_1) k_1(\alpha b_2) (\alpha r) i_2(\alpha r), \quad (1.10)$$

(ii) for $b_2 - b_1 \leq r \leq b_1 + b_2$

$$u(r) = \frac{A}{2\alpha^2 b_1 b_2} \{1 - \cosh[\alpha(b_2 - b_1)] e^{-\alpha r} - e^{-\alpha(b_1 + b_2)} \sinh(\alpha r)\}, \quad (1.11)$$

$$\begin{aligned} w(r) = & \frac{B}{16\alpha^4 b_1^2 b_2^2} \{2\alpha^2(b_1^2 + b_2^2) - 4 - \frac{3}{\alpha^2 r^2} [\alpha^4(b_2^2 - b_1^2)^2 + 4\alpha^2(b_1^2 + b_2^2) - 8] + \alpha^2 r^2 \\ & + 8\alpha r k_2(\alpha r) [(\alpha^2 b_1 b_2 - 1) \cosh \alpha(b_2 - b_1) + \alpha(b_2 - b_1) \sinh \alpha(b_2 - b_1)] \\ & - 8\alpha r i_2(\alpha r) e^{-\alpha(b_1 + b_2)} [\alpha^2 b_1 b_2 + \alpha(b_1 + b_2) + 1]\} \end{aligned} \quad (1.12)$$

(iii) for $r \geq b_1 + b_2$

$$u(r) = A i_0(\alpha b_1) i_0(\alpha b_2) (\alpha r) k_0(\alpha r), \quad (1.13)$$

$$w(r) = B i_1(\alpha b_1) i_1(\alpha b_2) (\alpha r) k_2(\alpha r). \quad (1.14)$$

The modified spherical Bessel functions $i_l(x)$ and $k_l(x)$ are given by

$$i_0(x) = \frac{\sinh x}{x}, \quad (1.15)$$

$$i_1(x) = \frac{x \cosh x - \sinh x}{x^2}, \quad (1.16)$$

$$i_2(x) = \frac{(x^2 + 3) \sinh x - 3x \cosh x}{x^3}, \quad (1.17)$$

and

$$k_0(x) = \frac{e^{-x}}{x}, \quad (1.18)$$

$$k_1(x) = \frac{e^{-x} (x + 1)}{x^2}, \quad (1.19)$$

$$k_2(x) = \frac{e^{-x} (x^2 + 3x + 3)}{x^3}. \quad (1.20)$$

To simplify the calculations using these wavefunctions, set $b_1 = b_2 \equiv b$. The coordinate space wavefunctions then become

(i) for $0 \leq r \leq 2b$

$$u(r) = \frac{A}{2\alpha^2 b^2} \{1 - e^{-\alpha r} - e^{-2\alpha b} \sinh(\alpha r)\} \quad (1.21)$$

$$w(r) = \frac{B}{16\alpha^4 b^4} \{-4(1 - \alpha^2 b^2) + \alpha^2 r^2 + \frac{24}{\alpha^2 r^2} (1 - \alpha^2 b^2) - 8(1 - \alpha^2 b^2)(\alpha r) k_2(\alpha r) - 8(1 + \alpha b)^2 e^{-2\alpha b} (\alpha r) i_2(\alpha r)\} \quad (1.22)$$

(ii) for $r \geq 2b$

$$u(r) = A i_0^2(\alpha b) (\alpha r) k_0(\alpha r), \quad (1.23)$$

$$w(r) = B i_1^2(\alpha b) (\alpha r) k_2(\alpha r), \quad (1.24)$$

The S-state and D-state probabilities are calculated using

$$P_S = \int_0^\infty k^2 dk |u(k)|^2, \quad (1.25)$$

$$P_D = \int_0^\infty k^2 dk |w(k)|^2, \quad (1.26)$$

with the result

$$P_S = \frac{A^2}{16\alpha^5 b^4} [8\alpha b - 9 + 4e^{-2\alpha b}(2\alpha b + 3) - e^{-4\alpha b}(4\alpha b + 3)], \quad (1.27)$$

$$P_D = \frac{B^2}{240\alpha^9 b^8} [56\alpha^5 b^5 - 135\alpha^4 b^4 - 80\alpha^3 b^3 + 450\alpha^2 b^2 - 315 - 60(\alpha b + 1)^2(2\alpha^3 b^3 + 3\alpha^2 b^2 - 7)e^{-2\alpha b} - 15(\alpha b + 1)^3(4\alpha^2 b^2 + 7\alpha b + 7)e^{-4\alpha b}]. \quad (1.28)$$

The normalisation constants A and B are, of course, related by the relation

$$P_S + P_D = 1. \quad (1.29)$$

Initial estimates, based on fitting the quadrupole moment and the root mean square radius show that $B^2 \approx 3.0A^2$ for a range parameter of $b = 1.475$ fm. Using $\alpha = 0.23165 \text{ fm}^{-1}$, then $\alpha b = 0.342$. This results in S-wave and D-wave probabilities of approximately 96% and 4%, respectively. The resulting values for the normalisation constants are $A = 0.905 \text{ fm}^{-1/2}$ and $B = 1.57 \text{ fm}^{-1/2}$ to 3 significant figures. The asymptotic normalisations are then

$$A_S = A i_0^2(\alpha b), \quad (1.30)$$

and

$$A_D = B i_1^2(\alpha b), \quad (1.31)$$

resulting in the values $A_S = 0.941 \text{ fm}^{-1/2}$ and $A_D = 0.0208 \text{ fm}^{-1/2}$. The D/S ratio is then $\eta = A_D/A_S = 0.022$. Figures 1 and 2 show $u(r)$ and $w(r)$ as compared with the NIJM I potential model calculations [3].

The root mean square radius is defined by

$$r_{rms} = \frac{1}{2} \sqrt{\int_0^\infty r^2 dr [|u(r)|^2 + |w(r)|^2]}, \quad (1.32)$$

and the quadrupole moment by

$$Q = \frac{1}{20} \int_0^\infty r^2 w(r) [\sqrt{8} u(r) - w(r)] dr. \quad (1.33)$$

Using Gaussian quadratures, one can easily perform the integrals numerically to obtain $r_{rms} = 2.08 \text{ fm}$ and $Q = 0.286 \text{ fm}^2$.

2. Conclusions

Spherical Bessel functions were used in non-local interactions describing the deuteron. It was shown that the resulting wavefunctions exhibit different behaviour in three different regions of coordinate space. This property is due to using the spherical Bessel functions. The agreement with the quadrupole moment of the deuteron was to 2 significant figures. More accuracy is expected when b_1 is set to be different from b_2 . Scattering calculations and the 3S_1 phase shifts are deferred to the next communication.

References

1. Y. Yamaguchi, Phys. Rev. 95 (1954) 1628.
2. Y. Yamaguchi and Y. Yamaguchi, Phys. Rev. 95 (1954) 1635.
3. V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen and J.J. de Swart, Phys. Rev. C49 (1994) 2950.